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TRIPLE REVERS EDERIVATION SONS EMIPRIME RINGS

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ABSTRACT

In this paper, we prove that if d is a Jordan reverse derivation of as emiprime ring R, then d Is a quarter reverse derivation. Using this, we show that d is a triple reverse derivation.

KEYWORDS: Semi Prime Ring, Jordan Reverse Derivation, Triple Reverse Derivation, Quarter Reverse Derivation, Center

INTRODUCTION

Herstein[1] studied Jordan derivations of prime rings and proved that every Jordan derivation on a prime ring of char.≠2 is a derivation and also studied reverse derivations of primering s.K. Suvarna and D. S. Irfana[3] studied some properties of Jordan derivations on semi prime rings.

We know that an additive mapping $d: R \to R$ is called a Jordan reverse Derivation if $d(x^2) = d(x)x + xd(x)$ for all x in R. An additive map d from a ring R to R is a triple reverse derivation if $d(xyx) = d(x)yx + xd(y)x + yxd(x)$ hold, for all x, $y \in R$ and d is a quarter reverse derivation if $d(xyzx) = d(x)xyz + xzd(y)x + d(z)xy + xzyd(x)$ hold, for all x, $y \in R$. Throughout this Paper, R will de not easemi prime ring and z its center.

Lemma 1: Let R be any ring and let $T(a) = \{r \in R / r(ax - xa) = 0, \text{ for al } lx \in R\}$, for all $a \in R$. Then $T(a)$is a two-sided ideal of R.

Proof: Clearly $T(a)$ is a left ideal of R. It remains to show that if $u \in T(a)$, $x \in R$, then $ux \in T(a)$. But then, for all $r \in R$, $u(axr - xra) = 0$.

Thus $u\{(ax - xa)r + x(ar - ra)\} = 0$.Since$u \in T(a), u(ax - xa) = 0$, and sowe have that $ux(ar - ra) = 0$, for all $r \in R$. Then $ux \in T(a)$. Hence the Lemma is proved.■

Lemma 2: If R is a prime ring and if $a \in R$ is not in Z, the center of R, then $T(a) = (0)$.

Proof: Since $a \notin Z$, for some $b \in R$, $ab - ba \neq 0$ .If $T(a) \neq (0)$, then $T(a)(ab - ba) = (0)$.So, the right ideal $T(a)$ is annihilated by an on-zeroelement. By the definition of a prime ring, either $T(a) = (0)$or $ab - ba = 0$.Since $a \notin Z$, $ab - ba \neq 0$. So,$T(a) = (0)$.■

Now, we prove the following results:

Theorem 1: If d is a Jordan reversed erivation of a semi prime ring R, then d is a quarter reverse derivation, that is, $d(abca) = d(a)abc + acd(b)a + ad(c)ab + acbd(a)$, for all $a, b, c \in R$.

Proof: We have $d(a^2) = d(a)a + ad(a)$

We replace a by $a + bc$. Then weget,

$\Rightarrow d(abc + bca) = d(abc) + d(bca)$

$\Rightarrow d(abc + bca) = d(c)ba + cd(b)a +$

$bcd(a) + d(a)cb + ad(c)b + cad(b),$ (1)

for all $a,\ b,\ c \in R$.

Similarly,

$\Rightarrow d(abc + bca) = d(c)ab + cd(b)a + cbd(a) +$

$d(a)bc + ad(c)b + acd(b), \ldots..$ (2)

for all $a,\ b,\ c \in R$

By equating equ.'s(1) and (2), weget

$\Rightarrow d(c)ba + bcd(a) +$

$d(a)cb + cad(b) = d(c)ab + cbd(a) + d(a)bc + acd(b), \ldots..$ (3)

for all $a, b, c \in R$

Consider $W = d((abc + bca)a + a(abc + bca))$

$= d(a)(abc + bca) + ad(abc + bca) + d(abc + bca)a + (abc + bca)d(a)$

From equ.(2)wehave,

$= d(a)abc + d(a)bca + a[d(c)ab + cd(b)a + cbd(a) + d(a)bc + ad(c)b + acd(b)] + [d(c)ab + cd(b)a + cbd(a) + d(a)bc + ad(c)b + acd(b)]a + abcd(a) + bcad(a)$

$\Rightarrow W = d(a)abc + 2d(a)bca + ad(c)ab + 2acd(b)a + acbd(a) +$

$ad(a)bc + a^2d(c)b + a^2cd(b) + d(c)aba + cd(b)a^2 +$ .. (4)$cbd(a)a + ad(c)ba + abcd(a) + bcad(a)$

On the other hand,

$\Rightarrow W = d((abc + bca)a + a(abc + bca))$

$\Rightarrow W = d(2abca + bca^2 + a^2bc)$

$= 2d(abca) + d(a^2)bc + a^2d(bc) + d(c)a^2b + cd(a^2b)$

$\Rightarrow W = 2d(abca) + d(a)abc + ad(a)bc + a^2d(c)b +$

$a^2cd(b) + d(c)a^2b + cd(b)a^2 + cbd(a)a + cbad(a)$ (5)

By comparing equ.'s (4) and (5), for *W*, we obtain,

$\Rightarrow 2d(a)bca + ad(c)ab + 2acd(b)a + acbd(a) + d(c)aba + ad(c)ba + abcd(a) + bcad(a) = 2d(abca) + d(c)a^2b + c$
$bad(a)$

From the equality given in equ.(3), we have,

$\Rightarrow 2d(a)abc + 2ad(c)ab + 2acd(b)a + 2acbd(a) = 2d(abca)$

$\Rightarrow d(abca) = d(a)abc + acd(b)a + ad(c)ab + acbd(a)$.

Theorem 2: If *d* is a Jordan reverse derivation of a semi prime ring *R*, then *d* is a triple reverse derivation, that is, $d(aba) = d(a)ba + ad(b)a + bad(a)$, for all $a, b, c \in R$.

Proof: We have $d(a^2) = d(a)a + ad(a)$

We replace a by $a+b$, then weget,

$\Rightarrow d(ab+ba) = d(ab) + d(ba)$

$= d(b)a + bd(a) + d(a)b + ad(b)$, for all $a, b, c \in R$. ….. (6)

Consider $W = d((ab+ba)a + a(ab+ba))$.

$= d(a)(ab+ba) + ad(ab+ba) + d(ab+ba)a + (ab+ba)d(a)$

$= d(a)ab + d(a)ba + a[d(b)a + bd(a) + d(a)b + ad(b)] + [d(b)a + bd(a) + d(a)b + ad(b)]a + abd(a) + bad(a)$

$= d(a)ab + d(a)ba + ad(b)a + abd(a) + ad(a)b + a^2d(b) + d(b)a^2 + bd(a)a + d(a)ba + ad(b)a + abd(a) + bad(a$

$\Rightarrow W = d(a)ab + 2d(a)ba + 2ad(b)a + 2abd(a) + ad(a)b +$

$a^2d(b) + d(b)a^2 + bd(a)a + bad(a)$….. . (7)

On the other hand, we get,

$\Rightarrow W = d((ab+ba)a + a(ab+ba))$

$= d(ba^2 + a^2b + 2aba)$

$= d(a^2)b + a^2d(b) + d(b)a^2 + bd(a^2) + 2d(aba)$

$\Rightarrow W = d(a)ab + ad(a)b + a^2d(b) +$

$d(b)a^2 + bd(a)a + bad(a) + 2d(aba)$….. (8)

By comparing equ.'s (7) and (8) for *W*, we obtain,

$\Rightarrow d(a)ab + 2d(a)ba + 2ad(b)a + 2abd(a) + ad(a)b + a^2d(b) + d(b)a^2 + bd(a)a + bad(a) = d(a)ab + ad(a)b + a$
$^2d(b) +$

$d(b)a^2 + bd(a)a + bad(a) + 2d(aba)$

$\Rightarrow 2d(a)ba + 2ad(b)a + 2bad(a) = 2d(aba)$

$\Rightarrow 2[d(a)ba + ad(b)a + bad(a)] = 2[d(aba)]$

$\Rightarrow d(a)ba + ad(b)a + bad(a) = d(aba)$■

We linearize the results of Theorem: 2 by replacing a by $a+c$, we arrive at

Theorem 3: For all $a,b,c \in R$,

$d(abc + cba) = d(a)bc + d(c)ba + ad(b)c + cd(b)a + bad(c) + bcd(a)$

Theorem 4: For all $a,b \in R$,$(d(a)b + ad(b))(ba - ab) - (ba - ab)d(ab) = 0$.

Proof: Consider $W = d(ab(ab) + (ab)ba)$

Byusing Theorem:3, with $c = ab$, we obtain,

$\Rightarrow W = d(ab)ab + (ab)d(ab) + d(ba)(ab) + bad(ab)$

$= d(ab)ab + (ab)d(b)a + (ab)bd(a) + d(a)b(ab) + ad(b)(ab) + bad(ab)$ (9)

However,$W = d(ab(ab) + ab^2a)$

$= d(ab(ab)) + d(ab^2a)$

$= d(ab)ab + (ab)d(ab) + d(b^2a)a + b^2ad(a)$

$\Rightarrow W = d(ab)ab + (ab)d(ab) + d(a)b^2a +$

$ad(b)ba + abd(b)a + b^2ad(a)$. (10)

W can also be written as,

$\Rightarrow W = d(ab(ab)) + d(ab^2a)$

$= d(ab)ab + (ab)d(ab) + d(a)ab^2 + ad(ab^2)$

$\Rightarrow W = d(ab)ab + (ab)d(ab) + d(a)ab^2 +$

$ad(b)ba + abd(b)a + ab^2d(a)$…. (11)

By equating equ.'s(10) and (11),weget,

$\Rightarrow d(a)b^2a + b^2ad(a) = d(a)ab^2 + ab^2d(a)$

$\Rightarrow d(a)b^2a = d(a)ab^2$and also, $b^2ad(a) = ab^2d(a)$ …. . (12)

By comparing equ.'s(9) and (10) for W, we obtain,

$\Rightarrow d(ab)ab+(ab)d(b)a+(ab)bd(a)+d(a)b(ab)+ad(b)(ab)+bad(ab)=d(ab)ab+(ab)d(ab)+d(a)b^2a+ad(b)ba+abd(b)a+b^2ad(a)$

Byequ.(12), wehave,

$\Rightarrow (ab)bd(a)+d(a)b(ab)+ad(b)(ab)+bad(ab)=(ab)d(ab)+d(a)b^2a+ad(b)ba+ab^2d(a)$

$\Rightarrow d(a)b(ab)+ad(b)(ab)+bad(ab)=(ab)d(ab)+d(a)b^2a+ad(b)ba$

By transposing and collecting terms, it follows that

$\Rightarrow (d(a)b+ad(b))(ba-ab)-(ba-ab)d(ab)=0$. ■

Theorem 5: If R is a semi prime ring of char. ≠2, then any Jordan reverse derivation of R is an ordinary reverse derivation of R. ■

Theorem 6: Let R beasemi prime ring of char.≠2and*d*isaJordanreversederivationof$R$.If*d*acts as an anti-homomorphism of R, then d is a central derivation.

Proof: Since d acts as ananti-homomorphism, we have,

$d(xy)=d(y)d(x)$ ….. (13)

From Theorem: 5, we have,

$d(xy)=d(y)x+yd(x)$, forall$x,y\in R$. Then

$\Rightarrow d(y)d(x)=d(y)x+yd(x)$ ….. (14)

By substituting zy for yi n equ.(14), we obtain,

$d(zy)d(x)=d(zy)x+zyd(x)$

$\Rightarrow d(zy)d(x)=d(y)d(z)x+zyd(x)$forall$x,y,z\in R$ ….. (15)

Since d is an anti-homomorphism.

On the other hand, we have,

$d(zy)d(x)=d(y)d(z)d(x)$

$=d(y)d(xz)$

$=d(y)[d(z)x+zd(x)]$

$\Rightarrow d(zy)d(x)=d(y)d(z)x+d(y)zd(x)$ ….. (16)

From equations (15) and (16), we have,

$d(y)d(z)x+zyd(x)=d(y)d(z)x+d(y)zd(x)$

$\Rightarrow zyd(x)=d(y)zd(x)$ ….. (17)

Put $y = xy$ in equ.(17)and using equ.(17)

$zxyd(x) = d(xy)zd(x)$

$= d(y)xzd(x) + yd(x)zd(x)$

$\Rightarrow zxyd(x) = zxyd(x) + yd(x)zd(x)$

$\Rightarrow yd(x)zd(x) = 0$

If were place z by ry, then weget,

$yd(x)ryd(x) = 0$, forall $r \in R$

$\Rightarrow yd(x)Ryd(x) = 0$

By semi primeness of R, $y\,d(x) = 0$.

$d(x) = 0$

Put $x = y$ implies $d(y) = 0$.

If we multiply with $[x,y]$ from the left hand side, then weget,

$[x,y]d(y) = 0$, forall $x,y \in R$ ….. (18)

We replace x by xz in equ.(18)and using equ.(18), then

$[xz,y]d(y) = 0$, for all $x, y, z \in R$

$\Rightarrow [x,y]zd(y) = 0$, forall $x, y, z \in R$ ….. (19)

On the otherhand,a linearization ofequ.(18)leads to

$[x,y+u]d(y+u) = 0$

$\Rightarrow [x,y]d(y) + [x,y]d(u) + [x,u]d(y) + [x,u]d(u) = 0$

$\Rightarrow [x,y]d(u) + [x,u]d(y) = 0$

$\Rightarrow [x,u]d(y) = -[x,y]d(u) = [y,x]d(u)$ ….. (20)

Ifwereplace z by $d(u)z[x,u]$ in equ.(19) and usingequ.(20), then we get,

$\Rightarrow [x,y]d(u)z[x,u]d(y) = 0$

$\Rightarrow -[x,y]d(u)z[x,y]d(u) = 0$

$\Rightarrow [x,y]d(u)z[x,y]d(u) = 0$ ….. (21)

Since R is semiprime, by equ. (21) weget, $[x, y]\,d(u) = 0$, for all x, y, u in

R. By[2] (i.e., Rings within volution–I.N.Herstein), $d(u) \in Z$, for all $u \in R$. This shows that d is a Jordan reversed erivation on R which maps R into its center. ■

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